

Sparsest cut and eigenvalue multiplicities on low degree Abelian Cayley graphs

Abstract

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Whether or not the Sparsest Cut problem admits an efficient $O(1)$ -approximation algorithm is a fundamental algorithmic question with connections to geometry and the Unique Games Conjecture.

Revisiting spectral algorithms for Sparsest Cut, we present a novel, simple algorithm that combines eigenspace enumeration with a new algorithm for the Cut Improvement problem.

The runtime of our algorithm is parametrized by a quantity that we call the cut dimension $CD(G)$: the smallest k such that the subspace spanned by the first k Laplacian eigenvectors contains all but a small fraction of a sparsest cut.

Our algorithm matches the guarantees of prior methods based on the threshold-rank paradigm, while also extending beyond them. To illustrate this, we study its performance on low degree Cayley graphs over Abelian groups---canonical examples of graphs with poor expansion properties.

Along the way to bounding the cut dimension of Abelian Cayley graphs, we prove a bound of $2^{O(d)}$ the multiplicity of the second smallest laplacian eigenvalue. This bound is tight and improves on a previous bound of $2^{O(d^2)}$ by Lee and Makarychev.

